

Worksheet 02 - Sigma Notation and Series

Expanding, evaluating and constructing finite and infinite series using sigma notation

Corrected notation: formulas use proper superscripts and subscripts, and sigma notation uses vertically stacked upper and lower limits.

Time	45 minutes
Total marks	35
Calculator	Allowed unless stated
Level	IB SL/HL mixed

Instructions: Show sufficient working for non-multiple-choice questions. Where appropriate, define variables, state restrictions and interpret results in context.

Section A - Multiple choice (5 marks)

1. The notation $\sum_{r=1}^5 r$ means:

- A. $1 + 2 + 3 + 4 + 5$
- B. $5 + 5 + 5 + 5 + 5$
- C. $r + r + r + r + r$
- D. $1 \times 2 \times 3 \times 4 \times 5$

Answer: _____

2. The first term of $\sum_{r=0}^4 (3r - 1)$ is:

- A. -1
- B. 0
- C. 2
- D. 3

Answer: _____

3. Which sigma expression represents $7 + 11 + 15 + 19$?

- A. $\sum_{r=1}^4 (4r + 3)$
- B. $\sum_{r=1}^4 (7r + 4)$
- C. $\sum_{r=0}^4 (7 + 4r)$
- D. $\sum_{r=1}^5 (4r + 3)$

Answer: _____

4. The series $-2 + 4 - 8 + 16 - \dots$ can begin with general term:

- A. $2(-2)^{r-1}$
- B. $-2(2)^{r-1}$
- C. $(-2)^r$
- D. $2r - 4$

Answer: _____

5. The value of $\sum_{r=1}^3 r(r + 1)$ is:

- A. 8
- B. 14

C. 20

D. 26

Answer: _____

Section B - Short answer (12 marks)

1. Expand and evaluate $\sum_{r=1}^5 (2r - 3)$. (3 marks)
2. Write the first five terms of $\sum_{r=1}^{\text{infinity}} (-1)^{r-1}/r$. (3 marks)
3. Write $6 + 16 + 26 + 36 + 46$ in sigma notation. (3 marks)

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4. A student writes $3 + 6 + 9 + 12$ as $\sum_{r=0}^4 3r$. Identify and correct the error. (3 marks)

Section C - Data response / case study (10 marks)

Case study: Training distances

A runner follows a 10-day plan. On day r , the planned distance in kilometres is $d_r = 2r + 1$.

Day r

1

2

3

4

5

...

10

Distance d_r

/ km

3

5

7

9

11

...

21

1. Write a sigma expression for the total planned distance over 10 days. (2 marks)

2. Calculate the total planned distance. (3 marks)

3. The runner changes the plan so that day 1 is still 3 km but the distance increases by 3 km each day.

Write the new d_r . (2 marks)

4. State one advantage and one limitation of using sigma notation in this context. (3 marks)

Section D - Extended response (8 marks)

Write a guide for a younger student explaining how to convert between a written series and sigma notation. Include one finite arithmetic example and one alternating example. Your explanation should mention the lower limit, upper limit and general term.

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Answer Key and Marking Guidance

Award marks for valid mathematical reasoning, clear notation and correctly interpreted results. Equivalent methods should receive full credit unless the question specifies a method.

Section A

1: A

2: A

3: A

4: A

5: C

Section B

1: $-1 + 1 + 3 + 5 + 7 = 15$. Award 1 for expansion, 1 for correct terms, 1 for sum.

2: 1, $-1/2$, $1/3$, $-1/4$, $1/5$.

3: $\sum_{r=1}^5 (10r - 4)$, or equivalent.

4: Using $r = 0$ to 4 gives five terms including $\sum_{r=1}^4 3r$ or $\sum_{r=0}^4 (3r + 0)$ and 12; correct is

3).

Section C

1: $\sum_{r=1}^{10} (2r + 1)$.

2: $3 + 5 + \dots + 21 = 120$ km.

3: $d_r = 3 + 3(r - 1) = 3r$.

4: Advantage: compact and general; limitation: may hide individual values or be hard to interpret without knowing the notation.

Section D

8 marks: 2 for explaining limits; 2 for identifying a valid general term; 2 for a correct finite arithmetic example; 1 for alternating sign handling; 1 for clarity and correct notation.