Worksheet 03 - Arithmetic Sequences and Series

Common difference, nth terms, arithmetic sums and linear modelling

Corrected notation: formulas use proper superscripts and subscripts, and sigma notation uses vertically stacked upper and lower limits.

Time	45 minutes
Total marks	35
Calculator	Allowed unless stated
Level	IB SL/HL mixed

Instructions: Show sufficient working for non-multiple-choice questions. Where appropriate, define variables, state restrictions and interpret results in context.

Section A - Multiple choice (5 marks)

1. An arithmetic sequence has a = 8 and d = -3. The 6th term is:

A. -7

B. -4

C. 5

D. 23

Answer: _____

2. For an arithmetic sequence, $u_n = 5n - 2$. The common difference is:

A. 2

B. 3

C. 5

D. 7

Answer: _____

3. The sum of the first n terms of an arithmetic sequence is:

A. $S_n = ar^{n-1}$

B. $S_n = n/2(2a + (n-1)d)$

C. $S_n = a/(1-r)$

D. $S_n = n^2 + d$

Answer: _____

4. The sequence 12, 17, 22, 27, ... first exceeds 100 at term:

A. 17

B. 18

C. 19

D. 20

Answer: _____

5. If $u_4 = 11$ and $u_9 = 31$ in an arithmetic sequence, d is:

A. 2

B. 4

C. 5

D. 6

Answer: _____

Section B - Short answer (12 marks)

1. Find the 20th term of the arithmetic sequence 7, 12, 17, 22, ... (2 marks)

2. An arithmetic sequence has $u_3 = 14$ and $u_8 = 39$. Find a and d. (4 marks)

3. Calculate the sum 4 + 10 + 16 + ... + 94. (3 marks)

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4. Find the smallest n such that the sum of 3 + 7 + 11 + ... up to n terms is greater than 500. (3 marks)

Section C - Data response / case study (10 marks)

Case study: Theatre seating

A theatre has 18 rows. Row 1 has 24 seats and each row after that has 3 more seats than the previous row.

Row

1

2

3

...

18

Seats

24

27

30

...

?

1. Write a formula for the number of seats in row r. (2 marks)

2. Find the number of seats in row 18. (2 marks)

3. Find the total number of seats in the theatre. (3 marks)

4. The theatre wants at least 1000 seats. How many extra rows of the same pattern are needed? (3 marks)

Section D - Extended response (8 marks)

Derive the formula $S_n = n/2(2a + (n - 1)d)$ for the sum of an arithmetic series. Then use it to find the sum of all positive integers less than 200 that are congruent to 2 modulo 5.

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Answer Key and Marking Guidance

Award marks for valid mathematical reasoning, clear notation and correctly interpreted results. Equivalent methods should receive full credit unless the question specifies a method.

Section A

1: A

2: C

3: B

4: C

5: B

Section B

1: $u_{20} = 7 + 19(5) = 102$.

2: $u_3 = a + 2d = 14$; $u_8 = a + 7d = 39$; $5d = 25$ so $d = 5$ and $a = 4$.

3: Terms: a=4, d=6, last=94. $n = (94-4)/6 + 1 = 16$. $S = 16/2(4+94) = 784$.

4: $S_n = n/2(2(3)+(n-1)4)=n(2n+1)$. Need $n(2n+1)>500$. n=15 gives 465; n=16 gives 528. Smallest n = 16.

Section C

1: $s_r = 24 + 3(r - 1) = 3r + 21$.

2: $s_{18} = 24 + 17(3) = 75$.

3: $S_{18} = 18/2(24 + 75) = 891$ seats.

4: Need $S_n \geq 1000$. $S_{19} = 19/2(24+78)=969$; $S_{20} = 20/2(24+81)=1050$. Two extra rows are needed.

Section D

8 marks: 3 for derivation by pairing first and last terms; 1 for formula; 2 for identifying terms 2, 7, 12, ..., 197 with n=40; 2 for sum = 40/2(2+197)=3980.